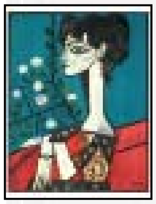
Maxwell's Dilemma



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Abstract:

Classical electromagnetic theory is applied to show that $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ predicts twice as much magnetic field strength for a current distribution than what is normally measured.

Since this above equation is one of Maxwell's equations which allegedly describes the propagation of light, the discussion questions the validity of the Uniform Plane Wave Equation (UPWE) as proposed by Maxwell.

Maxwell's Dilemma

1 Maxwell's Dilemma

The following derivation shows that Maxwell's equation predicts twice the magnetic field for a charge moving through space and a current traveling in a wire.

1.1 "Two much" flux

In this section we derive the total magnetic flux about a moving charge to show that Maxwell's equations predict too much flux by a factor of two; or "two much".

Consider a charge moving through space as shown in Figure 1-1

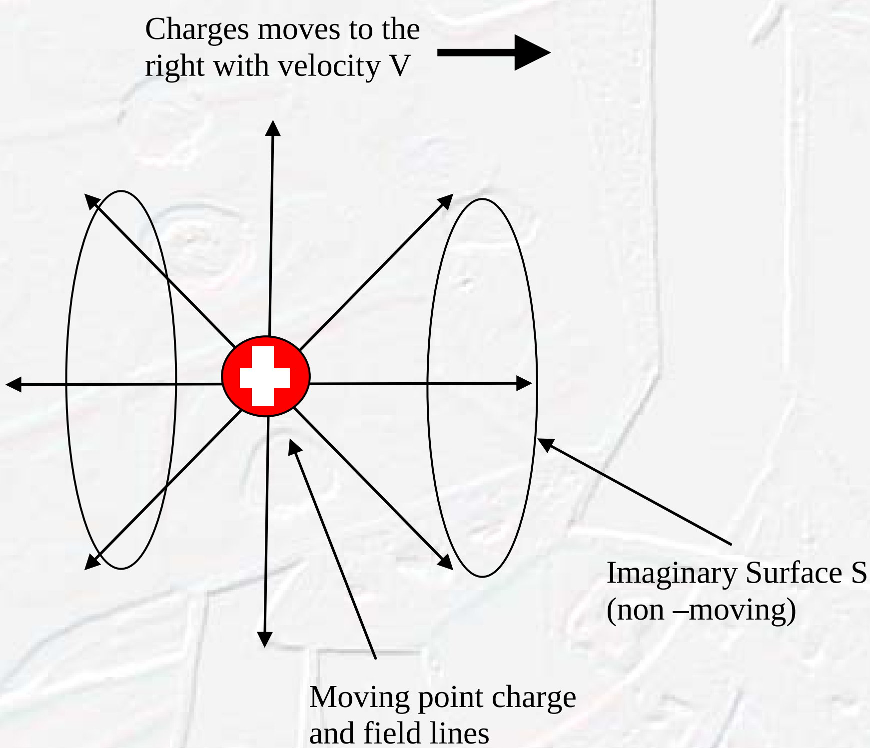

Figure 1-1: Moving charge

We already know that a moving charge will generate a magnetic field about itself due to the real charge. What about the displacement current? According to Maxwell's Equation, the magnetic field strength at the perimeter of an imaginary area is the summation of both the charge passing through the area and the displacement current effects. This is stated clearly in the following well known equation:

$$1)\ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Since the J term is easily calculated from Biot-Savart (see section 1.1.2 for more details) then what about the displacement current term?

From a conceptual standpoint we conclude that the displacement current term must also contribute a quantity of flux. If we were to place an imaginary surface (S) in front of the moving charge, then as the charge moves toward it, the concentration of the electric-flux passing through the surface increases. Since the dD/dt is positive and the direction of D is to the right, then an H field is created that will curl as shown in the following diagram. Next, if we were to place another imaginary surface behind the charge, then as the charge moves away from it, the concentration of electric flux lines through the surface decreases. The result is a negative dD/dt; however, since the direction of D is to the left then the direction of magnetic curl turns out to be in the same direction as the previous case.

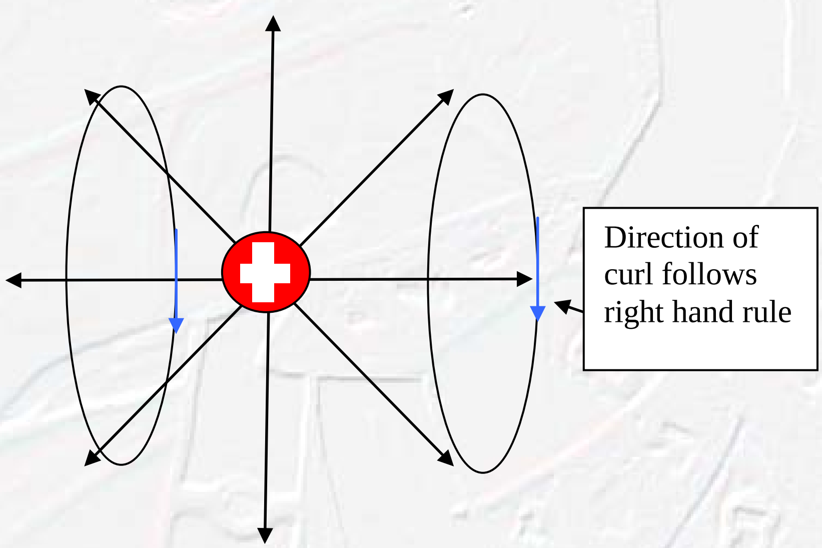

Figure 1-2: The Displacement magnetic field

With this in mind we next derive the general expression for displacement current induced magnetic field about a point charge moving through space.

1.1.1 Derivation of Displacement magnetic field about a point charge

In this section we derive the general expression for the magnetic field (B) of a charge (Q) due to the displacement current term of Maxell's equation. We start with:

$$2)\ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

The following diagram shows the parameters used in the derivation.

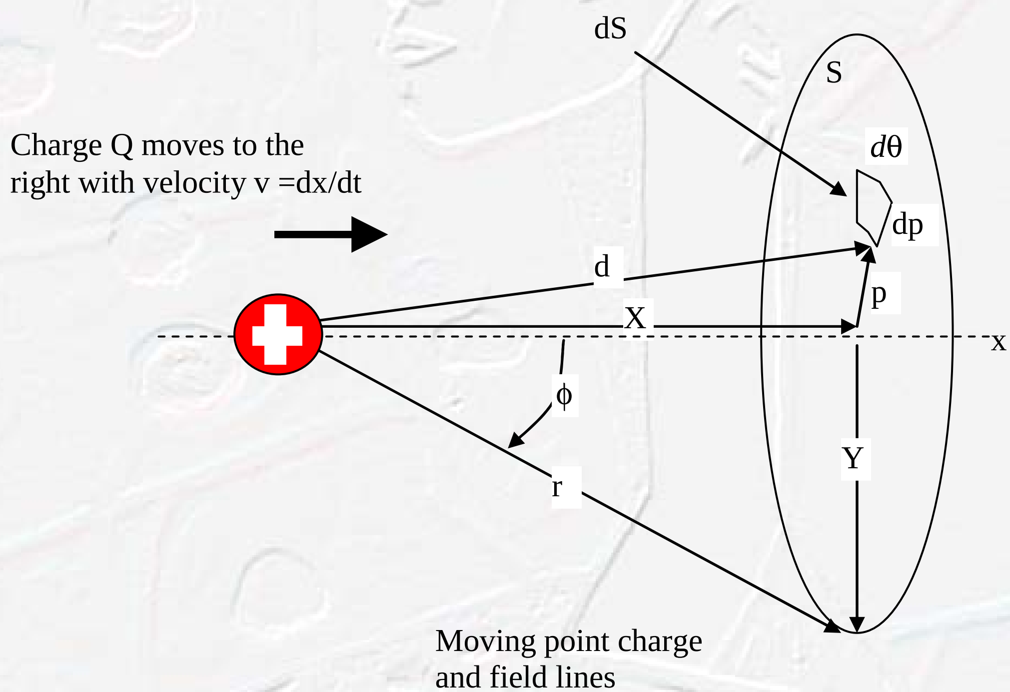

Figure 1-3: The variables used to parameterize the problem

Note: Lower case x is the x-axis and upper case X is the X component of distance from the charge to S. Y is the radius of S. The variables d and p parameterize dS.

Start by integrating both sides of step 2 over the surface of S.

$$3)\ \int_S (\nabla \times \mathbf{H}) \bullet dS = \int_S \frac{\partial \mathbf{D}}{\partial t} \bullet dS$$

Then applying Stoke's Theorem

4) $\oint_L \mathbf{H} \bullet d\mathbf{L} = \int_S \frac{\partial \mathbf{D}}{\partial t} \bullet d\mathbf{S}$ (L is the perimeter of S)

Since $\frac{d\Phi_E}{dt} = \int_S \frac{\partial \mathbf{D}}{\partial t} \bullet d\mathbf{S}$ then

5) $\frac{d\Phi_E}{dt} = \oint_L \mathbf{H} \bullet d\mathbf{L}$

Since H is perfectly uniform around L then

6) $\frac{d\Phi_E}{dt} = HL$ Then

7) $B = \frac{\mu}{L}\frac{d\Phi_E}{dt}$ Since $L = 2\pi Y$ Then

8) $B = \frac{\mu}{2\pi Y}\frac{d\Phi_E}{dt}$

Put step 8 aside for the moment. Next, develop an expression for the electric flux lines passing through S as a function of time.

Start with the well known Coulomb's Law in E form

9) $\mathbf{E} = \frac{Q\hat{\mathbf{d}}}{2\pi\varepsilon|\mathbf{d}|^2}$ Then

10) $\mathbf{D} = \frac{Q\hat{\mathbf{d}}}{2\pi|\mathbf{d}|^2}$ Since $\Phi_E = \int_S \mathbf{D} \bullet d\mathbf{S}$ then

11) $\Phi_E = \frac{Q}{4\pi}\int_{p=0}^{Y}\int_{\theta=0}^{2\pi}\frac{p\,d\theta\,dp}{\left(X^2 + p^2\right)}\hat{\mathbf{d}} \bullet \hat{\mathbf{x}}$

$\hat{\mathbf{d}} \bullet \hat{\mathbf{x}}$ is the cosine of the angle between d and x which is identical to $\frac{X}{d}$ or

$\hat{\mathbf{d}} \bullet \hat{\mathbf{x}} = \frac{X}{\sqrt{X^2 + p^2}}$ thus

12) $\Phi_E = \frac{Q}{4\pi} \int_{p=0}^{Y} \int_{\theta=0}^{2\pi} \frac{Xpd\theta dp}{\left(X^2 + p^2\right)^{3/2}}$

Integrate with respect to theta

13) $\Phi_E = \frac{QX}{2} \int_{p=0}^{Y} \frac{pdp}{\left(X^2 + p^2\right)^{3/2}}$ Then integrate with respect to p

14) $\Phi_E = \frac{Q}{2}\left[1 - \frac{X}{\sqrt{X^2 + Y^2}}\right]$

Now find the electric flux change as a function of X. This is done by differentiating both sides with respect to X.

15) $\frac{d\Phi_E}{dX} = \frac{Q}{2}\left[\frac{X^2}{\sqrt{\left(X^2 + Y^2\right)^3}} - \frac{1}{\sqrt{X^2 + Y^2}}\right]$

The above equation is the electric flux change as a function of the distance (X) between the charge and S. If the charge were moving to the right with velocity v, then the distance between the charge and S will be decreasing (–dX/dt); thus, using the chain rule of differential calculus we get

16) $\frac{d\Phi_E}{dt} = \frac{d\Phi_E}{dX}\frac{-dX}{dt} = \frac{Qv}{2}\left[\frac{1}{\sqrt{X^2 + Y^2}} - \frac{X^2}{\sqrt{\left(X^2 + Y^2\right)^3}}\right]$

Now substitute step 16 into step 8 and to get

17) $B = \frac{\mu}{2\pi Y}\frac{Qv}{2}\left[\frac{1}{\sqrt{X^2 + Y^2}} - \frac{X^2}{\sqrt{\left(X^2 + Y^2\right)^3}}\right]$

Recognizing the following

a) $|\mathbf{r}| = \sqrt{X^2 + Y^2}$

b) $\sin(\phi) = \dfrac{Y}{\sqrt{X^2 + Y^2}}$

c) $\cos(\phi) = \dfrac{X}{\sqrt{X^2 + Y^2}}$

d) $Y = |\mathbf{r}|\sin(\phi)$

Substitute a-d accordingly into 17

18) $B = \dfrac{\mu}{2\pi|\mathbf{r}|\sin(\phi)} \dfrac{Qv}{2|\mathbf{r}|}\left[1 - \cos^2\phi\right]$ Reducing and converting yields

19) $B = \dfrac{\mu Q v}{4\pi|\mathbf{r}|^2}\left[\sin\phi\right]$

From basic vector calculus $\hat{\mathbf{v}} \times \hat{\mathbf{r}} = \hat{\mathbf{n}}\sin\phi$ (where $\hat{\mathbf{n}}$ is a normal unit direction vector to v and r). Substitute to get

20) $B\hat{\mathbf{n}} = \dfrac{\mu Q \mathbf{v} \times \hat{\mathbf{r}}}{4\pi|\mathbf{r}|^2}$

Since B, by definition, is normal to (v x r) then we can simply write

21) $\mathbf{B} = \dfrac{\mu Q \mathbf{v} \times \hat{\mathbf{r}}}{4\pi|\mathbf{r}|^2}$

To make sure we do not confuse this derivation from others, apply the subscript D to signify that this magnetic field is from the displacement effects.

$$\mathbf{B}_D = \frac{\mu Q \mathbf{v} \times \hat{\mathbf{r}}}{4\pi|\mathbf{r}|^2}$$

Equation 1: Displacement magnetic field of moving charge

We will discuss the ramifications of this derivation in a moment.

In the next section we perform another quick derivation to give us something to compare to Equation 1.

1.1.2 The real magnetic field of a moving charge

The real magnetic field of a moving charge is determined from Biot-Savart

$$1)\ d\mathbf{B}_R = \frac{\mu I d\mathbf{L} \times \hat{\mathbf{r}}}{4\pi |\mathbf{r}|^2}$$

Realize that

$$Id\mathbf{L} = \frac{dq}{dt} d\mathbf{L} = dq \frac{d\mathbf{L}}{dt} = dq\mathbf{v}$$ (For v and dL in same direction)

Then substitute to arrive at

$$2)\ d\mathbf{B}_R = \frac{\mu dq\mathbf{v} \times \hat{\mathbf{r}}}{4\pi |\mathbf{r}|^2}$$

Integrate both sides with respect to dq to arrive at

$$\mathbf{B}_R = \frac{\mu Q\mathbf{v} \times \hat{\mathbf{r}}}{4\pi |\mathbf{r}|^2}$$

Equation 2: Real Magnetic field of moving charge

1.1.3 What does it mean?

According to Maxwell, a magnetic field is created by the summation of the real current and the displacement effects

$$1)\ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

This correlates to

$$2)\ \mathbf{B}_{total} = \mathbf{B}_R + \mathbf{B}_D$$ (for charges moving through space)

or

3) $$\mathbf{B}_{total} = \frac{\mu Q\mathbf{v}\times\hat{\mathbf{r}}}{4\pi|\mathbf{r}|^2} + \frac{\mu Q\mathbf{v}\times\hat{\mathbf{r}}}{4\pi|\mathbf{r}|^2} = \frac{\mu Q\mathbf{v}\times\hat{\mathbf{r}}}{2\pi|\mathbf{r}|^2}$$

This means that a charge moving thought space emits twice the magnetic field strength than is generally accepted.

This also means that current in a wire should produce twice the magnetic field strength than is normally attributed to Biot-Savart. I know there is someone who will claim (from knowledge of basic electricity) that there is no net electric field inside a good conductor and therefore Maxwell is saved; however, just because the NET electric field is zero does not mean that that $\frac{\partial \mathbf{D}}{\partial t} = 0$. This is easily proven by the following diagram:

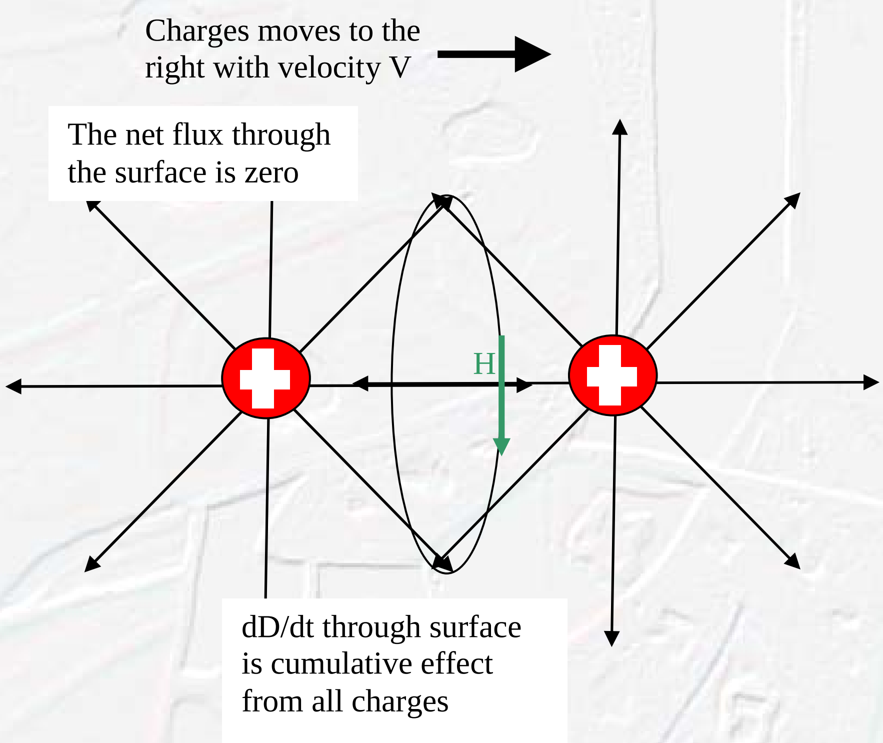

Figure 1-4: dD/dT is not zero even if D is.

If you did not quite follow the above, then take a closer look at the discussion around Figure 1-2.

Since we can not have twice the magnetic field emanating from wires containing real currents, then we must drop the displacement current term from Maxwell's equations. Thus

4) $\nabla \times \mathbf{H} = \mathbf{J}$

Or perhaps we compromise and suggest

5) $\nabla \times \mathbf{H} = \frac{1}{2}\mathbf{J} + \frac{1}{2}\frac{\partial \mathbf{D}}{\partial t}$

This compromise does not allow us to save Maxwell's Plane wave equation.

The only wave to save the plane wave equation is to suggest that

6) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$

The equation in step 6 can only be true if D is sourced by a conservative electric field. The only known source of a conservative electric field is a charge.

The E field in the other Maxwell Equation (see step 7 below) is not a conservative electric field.

7) $\nabla \times \mathbf{E}_M = -\frac{\partial \mathbf{B}}{\partial t}$

If the left side of 7 were a conservative electric field then it would violate Kirchhoff's Law ($\nabla \times \mathbf{E} = 0$). The E field in step 7 is not compatible to the D field of 6 (many free papers on this topic at www.distinti.com).

By specifying a magnetic field in terms of D might inadvertently give someone the false idea that charge is not required to convert an electric field to a magnetic field and vise versa. Therefore, we prefer to use the form that reminds us of this fact.

8) $\nabla \times \mathbf{H} = \mathbf{J}$

The preferred form?

Unfortunately step 8 is wrong because it says that there is only a magnetic field when the charge actually passes through the surface. We have shown that a circulation (H) about S will occur as the charge is approaching and as the charge leaves (it is true with B_D therefore it must be true with B_R). When we look back into our electromagnetism text books we find that $\nabla\times\mathbf{H}=\mathbf{J}$ is based on Ampere's circuital law which is inferred from an infinitely long current source. Am I the only engineer that "howls at the moon" when he sees a "point equation" which is only valid for an infinitely long current distribution? Thus, the equation in step 8 is only a good approximation under certain conditions (conditions which are never specified in any text book).

The wise thing to do is to completely toss out $\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$ and just say that a magnetic field is $\mathbf{B}_R=\frac{\mu Q\mathbf{v}\times\hat{\mathbf{r}}}{4\pi|\mathbf{r}|^2}$ or $\mathbf{H}=\frac{Q\mathbf{v}\times\hat{\mathbf{r}}}{4\pi|\mathbf{r}|^2}$.

According to New Electromagnetism; these equations are only valid for closed loop systems.

These changes start a snowball effect throughout classical electromagnetic theory.

2 Conclusion

Electromagnetism begins and ends with charges. This is written into the very definitions of the "fields" that we use:

E is a force per CHARGE field
H is AMPERE per meter field.
B and D are only different from the above by constants.

Each of these fields is created by forcing charges to behave a certain way. Each of these fields is measured by observing the behavior of charges caught within.

How space is disturbed by charges to create these fields and how fast these effects propagate through space is the big question. The only thing that is known for sure is that electric fields store potential energy and magnetic fields store kinetic energy (see New Induction ni.pdf). Without the presence of charge, it is impossible to convert one into the other. This is analogous to shooting a bullet into the sky of an airless planet. At the muzzle, the bullet will have kinetic energy. At a later point in time this kinetic energy is converted into potential energy as the bullet attains maximum altitude before falling back. This conversion between kinetic and potential energy can not occur without the medium of the bullet. Consequently, all wave phenomena rely on the transfer of energy from one form into the other; however, this transformation can not occur without a medium such as string or water. In an electric tank circuit (RLC), energy is alternately converted between kinetic energy (magnetic field) and potential energy (electric field); however, this occurs only because there are charges present to do this. Consequently, Maxwell's Uniform Plane Wave Equation (UPWE) was inferred from an electric tank circuit in which a medium of charges exists to allow the conversion between fields.

The UPWE is not valid for free space unless we conclude that free space contains a charge distribution. This could be the case; however, let us assume for the moment that it isn't. If there are no charges in space, then how does light propagate? We know that a charge can affect another distant charge through an electric or magnetic field. So why not suggest that light is either a purely electric or magnetic disturbance. Since New Induction is only an inverse law (not Inverse Square) its effects travel farther than other

New Electromagnetic effects; therefore, it is the most likely candidate for explaining electromagnetic radiation. A dipole derivation (based on New Induction) for both transverse and longitudinal effects is found in the book NIA1.