



A classical attempt at self-inductance



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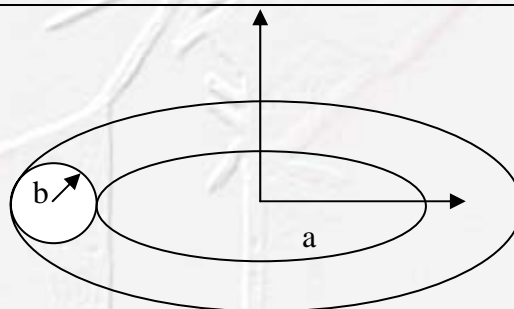
The Loop-Inductance equation found in Electrodynamics 3ed by Jackson

In chapter 5 of the book Classical Electrodynamics (3rd Ed) by Jackson there is allegedly a solution for the inductance of a single turn loop. The final expression is shown in equation A3.

$$A1) L_{SELF} = \mu_0 a \left[\ln\left(\frac{8a}{b}\right) - 2 \right]$$

$$A2) L = L_{SELF} + L_{INTRINSIC}$$

$$A3) L = \mu_0 a \left[\ln\left(\frac{8a}{b}\right) - \frac{7}{4} \right]$$



In the Anomalies of Classical Electromagnetism paper (apoce.pdf) it is stated that it is impossible to derive inductance from Classical Electrodynamics. This position still holds since the main component of Equation A3 is the self-inductance of the loop (Equation A1) which is computed from vector magnetic potentials. The result Jackson obtains for Equation A1 is actually the mutual induction between two filamentary loops.

This paper provides two independent proofs which show that Equation A1 is actually the mutual induction between two loops rather than the self-induction of a single loop.

The first proof is found in James Clerk Maxwell's Treatise on Electricity and Magnetism Volume 2, Article 704. Maxwell derived the mutual inductance between two loops using the exact same techniques as Jackson and obtains the exact same expression.

For the second proof, Jackson's derivation of Equation A1 is paralleled using New Electromagnetism to show that the expression is actually an approximation for the mutual inductance between two filamentary loops.

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1 Introduction

This paper shows two different proofs that Jackson's derivation for the induction of a single turn loop is actually the mutual induction between two loops.

The first proof shows that Maxwell's derivation for the mutual inductance between two loops is identical to what Jackson supposedly derives for the self-induction of a single-turn loop.

The second proof shows the New Electromagnetism interpretation of Jackson's result by showing side-by-side derivations using New Electromagnetism and the Classical theory as applied by Jackson. The New Electromagnetism derivation agrees with the Maxwell result shown in the first proof.

We do not know if Jackson is the originator of the derivation for problem 5.32; however, we will call it Jackson's derivation throughout this text for the sake of convenience.

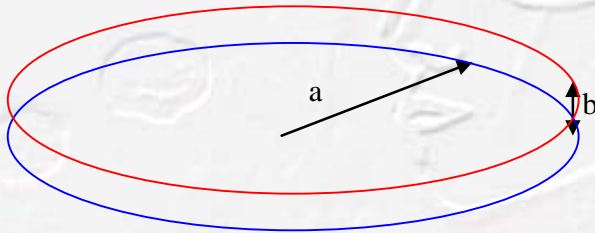
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2 The Proof using Maxwell

In James Clerk Maxwell's Treatise on Electricity and Magnetism Vol 2. (Article 704), the mutual induction between two loops (A4) is derived using elliptical integrals.

$$A4) M = \mu_0 a \left[\ln\left(\frac{8a}{b}\right) - 2 \right]$$



This results is identical to the expression for the self-inductance (A1) reported by Jackson. Experimental data found later in this paper shows that Jackson's predicted value for single turn loop inductors are not very good.

To be thorough, we can obtain Jackson's final result (A3) by adding intrinsic inductance to Maxwell's results (A4). To demonstrate this, we compute the value for the intrinsic-inductance of a single turn loop.

The accepted model for intrinsic-inductance is $\frac{\mu}{8\pi}$ (H/m).

Since the length of the loop is $2\pi a$, we get a total intrinsic-inductance component of $\frac{\mu a}{4}$ Henries. When this result is added to Maxwell's expression (A4), we get the same answer as Jackson which is:

$$A5) \mu_0 a \left[\ln\left(\frac{8a}{b}\right) - \frac{7}{4} \right].$$

The summation of the Mutual Induction between two loops plus intrinsic inductance of a single turn loop is meaningless.



This is the first of two proofs which show that Jackson's result is meaningless. The next proof shows the step-by-step derivation of Jackson's result compared with the New Electromagnetism meaning. Before delving into the derivation we must ask some basic questions.



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3 Questions

Before we present the step-by-step derivation found in Jackson, we present some intriguing questions in order to put your mind in the proper frame of thought.

Jackson used magnetic potentials to derive an approximation for the inductance of a single turn loop. We offer these questions in the beginning to give you something to think about during the derivation.

The following questions are not intended to “disprove” magnetic potentials.

These questions will be answered in the text that follows.

3.1 What is Magnetic Potential?

Magnetic potentials are a mathematical construct developed to give the science of magnetism a tool which is analogous to voltage. There are two types of magnetic potentials: Vector Magnetic Potentials and Scalar Magnetic Potentials. We will provide a brief intro to magnetic potentials in the next chapter.

Because magnetic potentials are only mathematical constructs, they can not be verified by experiment. It is asserted that the Aharonov-Bohm experiment proves the existence of Vector Magnetic Potentials; however, we will publish another paper which shows that the explanation of the Aharonov-Bohm effect is also an improper application of Vector Magnetic Potentials. In fact, the effect can be explained in terms of the fundamental force equations of New Magnetism.

3.2 Why does it give an answer?

How can the Vector Magnetic Potential (VMP) method (as found in Jackson) give us an answer to the induction problem where the fundamental Maxwell equations do not?

The VMP is only an abstraction of Maxwell’s equations; therefore, how can it explain more?

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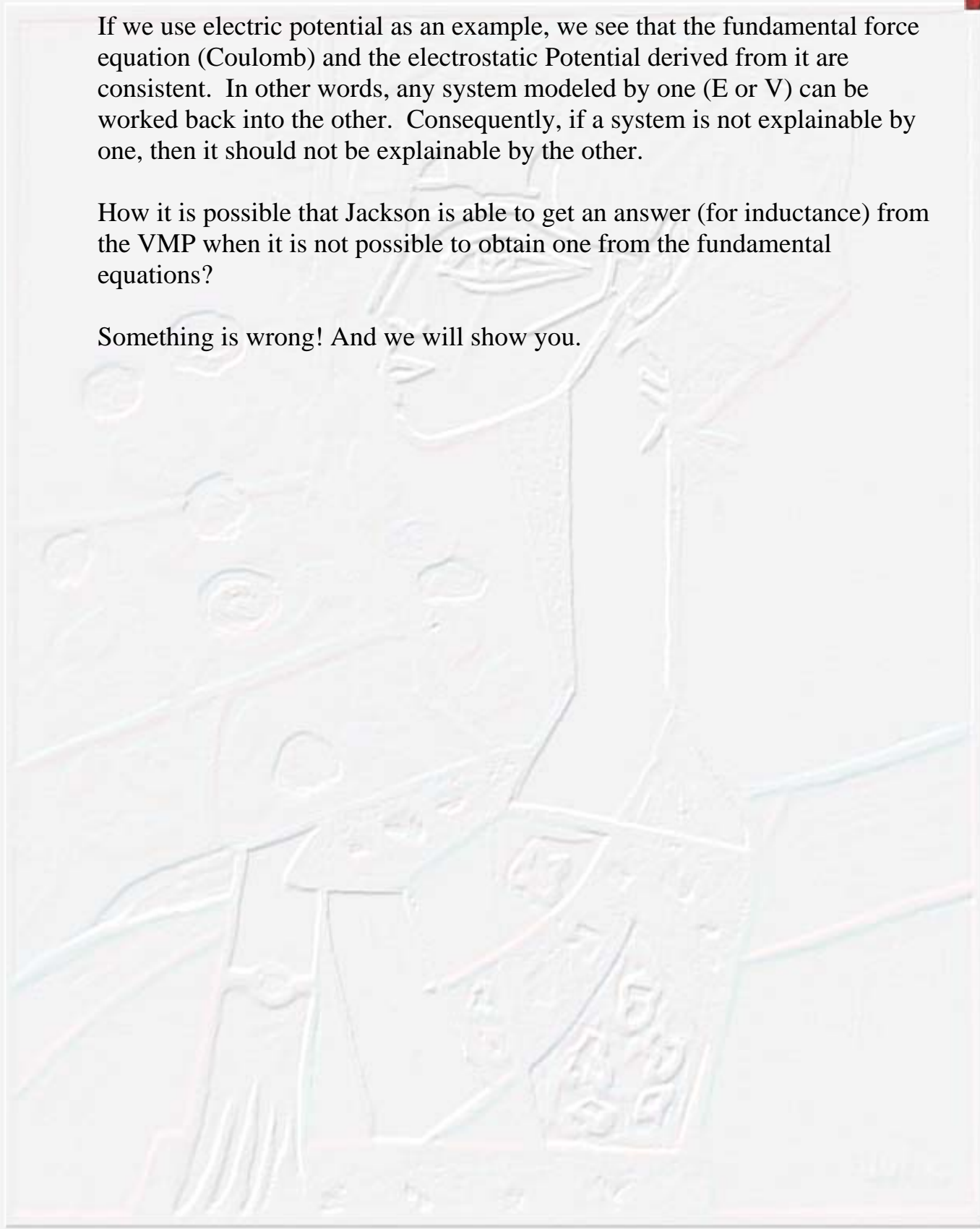


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If we use electric potential as an example, we see that the fundamental force equation (Coulomb) and the electrostatic Potential derived from it are consistent. In other words, any system modeled by one (E or V) can be worked back into the other. Consequently, if a system is not explainable by one, then it should not be explainable by the other.

How it is possible that Jackson is able to get an answer (for inductance) from the VMP when it is not possible to obtain one from the fundamental equations?

Something is wrong! And we will show you.





4 Magnetic Potentials

There are actually two types of magnetic potentials: vector magnetic potentials (VMP) and scalar magnetic potentials (SMP).

The SMP is just a negated Ampere's Circuital Law where the closed loop restriction is removed as such

$V_{M,ab} = -\int_a^b \mathbf{H} \cdot d\mathbf{L}$ has the units of "amps", but where those amps are is ambiguous (and thus violates the ambiguity rule of Rules of Nature (ron.pdf)).

The VMP is a more convoluted derivation but ends up as such:

$\mathbf{A} = K_M \oint \frac{Id\mathbf{L}}{R}$ Where $K_M = \frac{\mu}{4\pi}$ and A is flux/meter such that

$$\Phi = \oint \mathbf{A} \cdot d\mathbf{L}$$

Magnetic potentials were developed to provide an analog to Electric potential. With that said, here are some important differences:

	Electric Potential	Magnetic Potential
Units	Measures potential energy per charge (Volts)	Does not represent potential energy.
Measurability	Can be directly measured. Many instruments read out in volts (DVM, Scopes, etc)	Can not be directly measured. No device is made (that we are aware of) which reads out in magnetic potential. There is no experiment which can prove that magnetic potentials exist.
Acts directly on charges	Y	N
Basis for circuit theory	Used in electric circuit theory	Not used in magnetic circuit theory
Vector	Not a vector	There is both a scalar and vector version.



Because VMP is used in the Jackson derivation, VMP are discussed further.

4.1 The Vector Magnetic Potential

4.1.1 The classical version

In classical electrodynamics the vector magnetic potential is defined by the following:

$$1) \mathbf{A} = K_M \oint \frac{I d\mathbf{L}}{R} \quad \text{where } K_M = \frac{\mu}{4\pi}$$

A is only a partial result; therefore, in order to make use of A it must be integrated around a closed loop as such.

$$2) \Phi = K_M \iint_S \iint_T \frac{I_S d\mathbf{L}_S \cdot d\mathbf{L}_T}{r}$$

The above equation only requires a few more operations to become the Neumann equation. In our paper ni_neumann.pdf we show quite exhaustively that classical theory requires that the integration of the vector magnetic potential must be closed otherwise it would be describing the spherical field of New Electromagnetism.

4.1.2 The New Electromagnetic Version

We can derive a New Electromagnetic equivalent of vector magnetic potentials from the New Induction equation

New Induction (v3)

$$1) V_K = -K_M \iint_S \int \left(\frac{dI_S}{dt} \right) \frac{d\mathbf{L}_S \cdot d\mathbf{L}_T}{|\mathbf{r}|}$$

Then drop the dLT

$$2) \mathbf{M} = -K_M \int_S \left(\frac{dI_S}{dt} \right) \frac{d\mathbf{L}_S}{|\mathbf{r}|}$$

Integrate both sides by time and multiply by -1 to yield

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$$3) - \int M dt = K_M \int_S \frac{IdL_S}{|\mathbf{r}|}$$

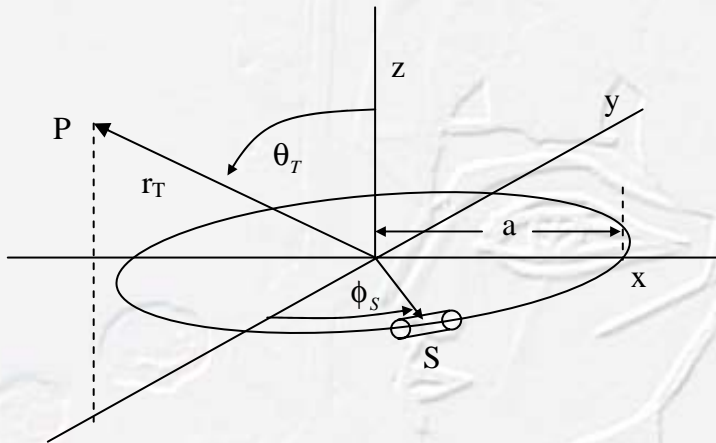
The right hand side of the equation is now identical to the classical definition of the vector magnetic potential except that it is not constrained to closed loops. You will notice that the left hand side is just as obscure as the classical definition for vector magnetic potential. Therefore, we will use the equation in step 2 for the New Electromagnetic representation for a vector potential. The M-Field (see ne.pdf) is similar to an electric field in that it has the units of force-per-coulomb; the major difference is that it is not conservative.

$$\mathbf{M} = -K_M \int_S \left(\frac{dI_S}{dt} \right) \frac{d\mathbf{L}_S}{|\mathbf{r}|}$$

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5 The Induction Problem



The above diagram is similar to the one used by Jackson on page 182 except that we are using New Electromagnetism notation (see ne.pdf for detailed list) which is less confusing.

The induction problem requires a number of steps before a value for induction is actually derived.

5.1 The first step

In the first step we are required on page 182 to find the Vector Magnetic potential at point P about a “Filamentary” current source.

Jackson arrives at the following in equation 5.36 on page 182

$$J1) A\phi(r, \theta) = K_M I_S a \int_0^{2\pi} \frac{\cos \phi_S d\phi_S}{\sqrt{a^2 + r_T^2 - 2a r_T \sin \theta_T \cos \phi_S}} \quad (\text{Jackson (5.36)})$$

Applying the New Electromagnetic version found in section 4.1.2

$$NE1) M\phi(r, \theta) = -K_M \frac{dI_S}{dt} a \int_0^{2\pi} \frac{\cos \phi_S d\phi_S}{\sqrt{a^2 + r_T^2 - 2a r_T \sin \theta_T \cos \phi_S}}$$

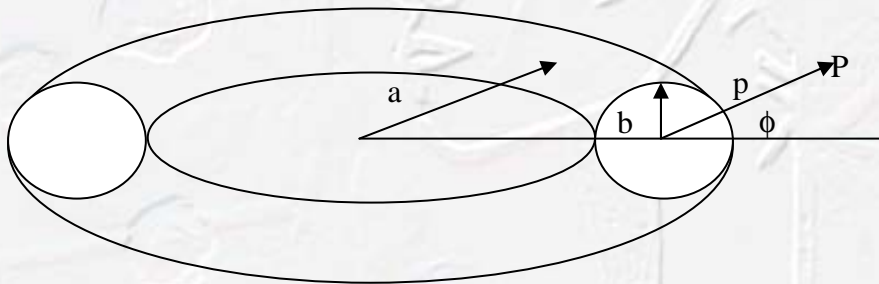
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The above integrations are solved with elliptic integrals to provide decent Approximations. For more information on elliptic integral, see the CRC Standard Math Tables which can be purchased through one of the links found on the page where this paper is downloaded.

5.2 The Second Step

In the second step, we are to determine the vector magnetic potential at a point near the surface of a ring using the “Filamentary” approximation for vector magnetic potential from the first step.



In the above, a is much much larger than b .

Jackson arrives at

$$J2) A_{\phi} = 2K_M I_S \left[\ln\left(\frac{8a}{p}\right) - 2 \right] \text{ (Jackson (problem 5.32a))}$$

Applying The New Electromagnetic version

$$NE2) M_{\phi} = -2K_M \frac{dI_S}{dt} \left[\ln\left(\frac{8a}{p}\right) - 2 \right] \text{ (M-Field intensity along Phi direction)}$$

I ask you, which equation gives you a clearer picture of what is really going one here? The Vector Magnetic Potential or the M-Field.

The New Electromagnetism equation states that there is a force-per-charge (M-Field) at point P that kicks charges in the opposite direction to the current change at the center of the wire.

At this point we continue with Jackson and get back to New Electromagnetism later.



5.3 Jackson 3rd Step (5.32b)

In the third step Jackson requires us to determine an expression for the vector magnetic potential at a point inside of the wire. The purpose of this step is to account for intrinsic inductance of the wire.

The result obtained by Jackson is

$$J3) A_{\phi} = K_M I_S \left[1 - \frac{p^2}{b^2} \right] + 2K_M I_S \left[\ln\left(\frac{8a}{b}\right) - 2 \right] \text{ for } p < b$$

This step affects the result by very little as will be demonstrated in the fourth and final step.

5.4 Jackson 4th step (5.32c)

In the fourth step we are required to use equation 5.149 (from Jackson's book) in order to solve for the magnetic energy and hence the self inductance.

$$\text{Equation 5.149 is } W = \frac{1}{2} \int \mathbf{J} \cdot \mathbf{A} d^3x$$

And the result Jackson gets is

$$J4) L = K_M 4\pi a \left[\ln\left(\frac{8a}{b}\right) - \frac{7}{4} \right]$$

Notice that the intrinsic inductance accounted for in step 3 only changed the 2 to 7/4. This result is consistent with the results obtained using Maxwell's derivation found previously.

5.5 New Electromagnetism Solution

The New Electromagnetism solution is easier to follow.



We simply observe that a forward accelerating current in a filamentary loop generates a “Backward M-field”. If we had a second loop parallel to the first loop such that the distance between the loops is much smaller than the radius of the loops, then we can compute the V_K coupled to the second loop from the expression

$$V_K = \int \mathbf{M} \cdot d\mathbf{L} .$$

Since the M-Field will be uniform at the second loop (because it is parallel to the first), thus we can simply compute V_K from the following

$$V_K = |\mathbf{M}| * \textit{perimeter}$$

Thus, simply multiply (NE2) by the perimeter of the ring and set p to b; thus:

$$\text{NE3) } V_K = -K_M 4\pi a \frac{dI_s}{dt} \left[\ln\left(\frac{8a}{b}\right) - 2 \right]$$

Mutual Induction is found from the following expression

$$M = -\frac{V_K}{\frac{di}{dt}} \text{ (Mutual Inductance)}$$

Thus,

$$\text{NE4) } M = K_M 4\pi a \left[\ln\left(\frac{8a}{b}\right) - 2 \right] .$$

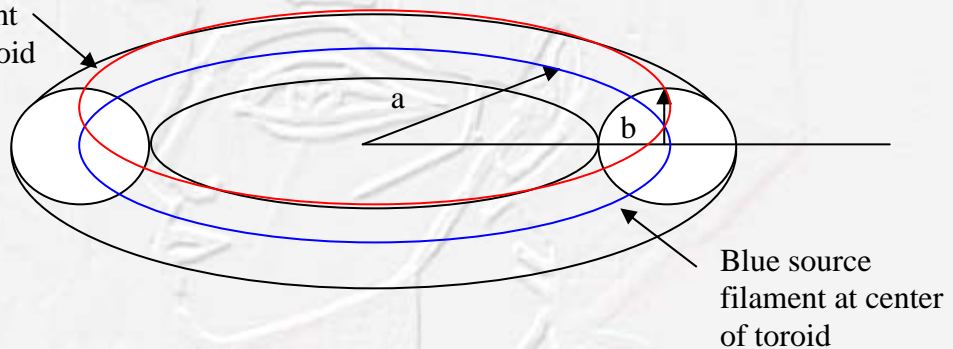
Of course, this does not account for intrinsic inductance as Jackson does; however, it makes no sense to add intrinsic inductance of a loop to the mutual inductance (M) between two loops.



6 What did we really do?

We went on a long drawn out adventure to calculate the mutual inductance between a filamentary current ring at the center, and another located at the edge. This is shown more clearly by the following diagram.

Red target filament at periphery of toroid



Blue source filament at center of toroid

Therefore, we did not solve for inductance; instead, we developed an approximation by calculating the mutual inductance between two loops.

The New Induction approach is easier to follow. It also represented more clearly what is happening.

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7 Conclusions

There are multiple conclusions that we can draw from this exercise. We have a subchapter for each conclusion.

7.1 This is not Inductance

It should be clear to the reader that the values for inductance derived in the methods presented in this paper are not true representations of the inductance of a single turn loop of wire. The equations are in fact approximations for the mutual inductance between two filamentary loops of wire.

In section 7.3 we will explain how New Electromagnetism solves the self induction problem.

For a “two-loop” approximation, the results obtained are not too bad. The following table compares Jackson to experimental results:

48 inch perimeter loops	Measured inductance nH	Jackson nH
22 AWG wire	2055	1611
26 AWG wire	2655	1724

With more experiment, it might be possible to develop a correction factor that could adjust Jackson to experimental results. We leave this exercise to the motivated reader.

7.2 Vector Magnetic Potentials

As stated in the opening section of this document, Vector Magnetic Potentials are confusing. Not only are they contradictory in definition, they actually hide the true essence of what is being derived (as demonstrated in this paper).

In fact, the derivation found in Jackson should have set off alarm bells years ago. How can vector magnetic potentials describe more than the basic models from which it was derived? How come nobody questioned this



blatant contradiction? To say it another way: if it is not possible to derive a relationship for self-induction from the fundamental electromagnetic force equations (Maxwell, Faraday, et al), then it can not be possible to do it from an abstraction of those same models (such as Vector Magnetic Potentials).

The paradox is resolved by realizing that the results of the derivations are not self inductance; instead, we did an awful lot of math to solve a stupid mutual inductance problem.

In any event, New Electromagnetism does away with Vector Magnetic Potentials. There are no applications which traditionally required Vector Magnetic potentials that can not be solved with the New Electromagnetic fundamental force equations. As demonstrated in this paper, the New Induction solution was simpler and easier to follow. It enabled us to see more clearly what was being derived.

False Advertising

It is claimed that the Jackson's derivation is for a loop with a uniform current density in the cross section. If this is so, then why does the derivation hinge upon a filamentary current distribution localized at the core of the wire?

7.3 What is the solution

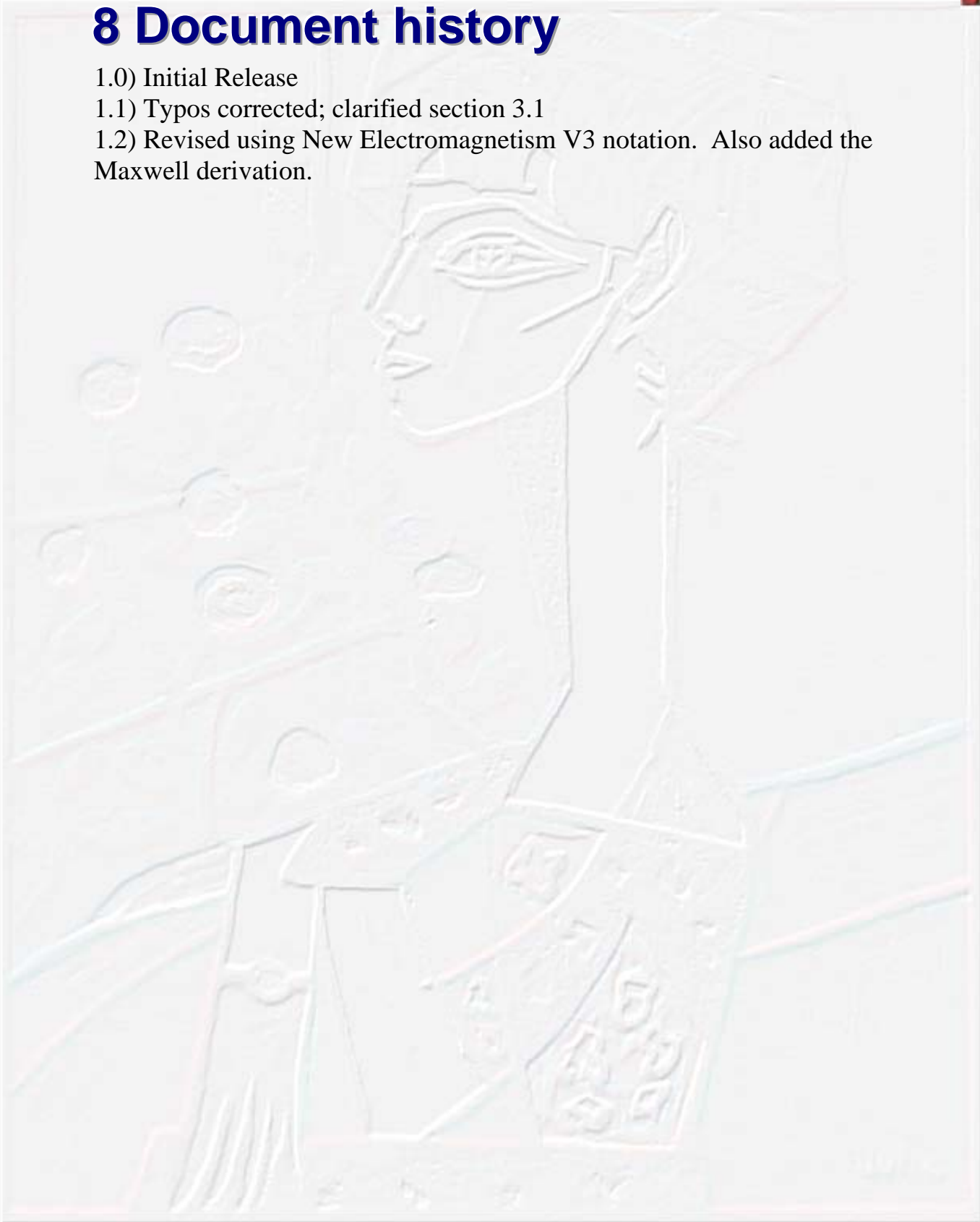
The actual solution to self-induction is much more complicated than the attempt presented in Jackson. Simply applying New Induction to a filamentary loop will not do either. The problem requires breaking a wire down in to finite volumes. Each volume contains a certain number of free and excess carriers (See New Magnetism) whose motions (velocity, acceleration and concentration) all must be accounted for.

Such a solution can only be handled by computer and requires intense computer time. In order to reduce computer time without loss in accuracy, we have developed proprietary adaptive integration algorithms that can reduce the computation time of such complex interactions by orders of magnitude. The development is the subject of my graduate thesis to be published shortly.



8 Document history

- 1.0) Initial Release
- 1.1) Typos corrected; clarified section 3.1
- 1.2) Revised using New Electromagnetism V3 notation. Also added the Maxwell derivation.



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